

Holographic Nuclei : Supersymmetric Examples

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ABSTRACT: We provide a dual gravity description of a supersymmetric heavy nucleus, following the idea of our previous paper arXiv/0809.3141. The supersymmetric nucleus consists of a marginal bound state of A baryons distributed over a ball in 3 dimensions. In the gauge/string duality, the baryon in $\mathcal{N} = 4$ super Yang-Mills (SYM) theory corresponds to a D5-brane wrapping S^5 of the $AdS_5 \times S^5$ spacetime, so the nucleus corresponds to a collection of A D5-branes. We take a large A and a near horizon limits of a back-reacted geometry generated by the wrapped A D5-branes, where we find a gap in the supergravity fluctuation spectrum. This spectrum is a gravity dual of giant resonances of heavy nuclei, in the supersymmetric toy example of QCD.

1. Gauge/string duality and heavy nuclei

Heavy nuclei are bound states of large number of nucleons. Despite their importance in physics, a fundamental understanding of it based on QCD is still missing. This is partly because of complication of the many-body system, where mass number A of a nucleus is large. The gauge/string duality [1, 2] (the AdS/CFT correspondence) opened a new path to solve strongly coupled gauge theories such as large N QCD, in which essentially a large number of D-branes are necessary to validate the dual gravity description. Baryons generically correspond to, in the gravity description of large N QCD-like theories, D-branes wrapping nontrivial cycles (called baryon vertices) [3]. In our previous paper [4], we noticed that, *we can take a large A limit and a near horizon limit of the wrapped D-branes for extracting the collective physics of A baryons, i.e. physics of a heavy nucleus*. This procedure is equivalent to looking at a particular sub-sector of the holographic QCD. This idea was initiated in our previous paper [4], and a schematic picture of the procedure is shown in Fig. 1.

In this paper, we provide supersymmetric examples realizing this idea. In string theory, supersymmetric setups are more familiar, and our example in this paper is for the $\mathcal{N} = 4$ supersymmetric Yang-Mills (SYM) theory for which the original gauge/string duality was conjectured. We will find a near horizon geometry produced by a large number of baryon vertices (A D5-branes wrapping the S^5 with $A \rightarrow \infty$). Fluctuation analysis of the background geometry provides, through the gauge/string dictionary, a spectrum of a collective motion of the baryons, *i.e.* giant resonances of a heavy nucleus with mass number A .

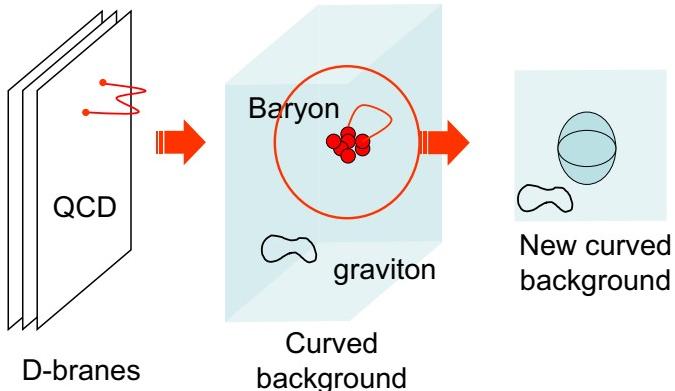


Figure 1: How to obtain gravity backgrounds dual to the heavy nuclei. The left arrow means taking a geometry dual to a large N QCD. Then one has “baryon vertices” (= particular D-branes wrapping cycles) in the geometry. The right arrow shows the procedure studied in this paper. We consider further a near horizon geometry of the back-reacted geometry produced by the baryon vertices. This corresponds to looking at a sub-sector of the whole holographic QCD, which describes collective excitations on the heavy nucleus.

2. Geometry around baryon vertices

Let us start by reviewing basic properties of the baryon vertex [3] in the $AdS_5 \times S^5$ geometry in type IIB superstring theory. This geometry is a gravity description of the $\mathcal{N} = 4$ SYM in the large N and the large 'tHooft coupling limits, and the baryon corresponds to a D5-brane wrapping the S^5 and localized in the radial direction of the AdS_5 as $r = r_0$, called “baryon vertex”.

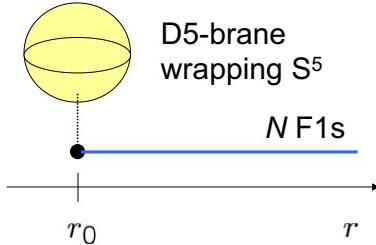


Figure 2: The baryon vertex.

The reason why the D5-brane corresponds to a baryon in the boundary field theory is as follows. The effective action of the D5-brane has a Chern-Simons term of the form

$$\int_{S^5 \times R} C_4 \wedge F \sim \int dC \wedge A_0 dx^0, \quad (2.1)$$

meaning that the background Ramond-Ramond 5-form flux $dC_4 \propto N d\Omega^{(S^5)}$ in the geometry generates N electric charges on the D5-brane. The charge provides an electric flux going out from the D5-brane, which is N fundamental strings (see Fig. 2), which are interpreted as (infinitely massive) N quarks in the SYM.

One can solve the BPS equations of motion of the D5-brane action [5], and can see that indeed the D5-brane configuration is supersymmetric. The fundamental strings attached to it is represented by a spike configuration [6]. According to the scale-radius correspondence of the gauge/string duality, the size z_0 of the baryon measured in the boundary SYM theory is related to r_0 , the position of the D5-brane in the AdS_5 . Borrowing a similar interpretation of a SYM instanton size in terms of the bulk location of a D(-1)-brane (for example see [7]), we have

$$z_0 = \alpha' \sqrt{2\lambda} r_0^{-1} \quad (2.2)$$

where λ is the 'tHooft coupling of the SYM. It was found [5] that the total energy of the baryon vertex does not depend on r_0 , and is equal to that of N fundamental strings extending from the AdS horizon to AdS boundary: $E_{D5 \text{ spike}} = N E_{F1}$.

Now, let us turn to the nucleus. Any nucleus is a collection of A nucleons, so it is just A D5-branes accumulated. Therefore, the low energy theory on the nucleus is a non-Abelian D5-brane effective theory, which is a $U(A)$ SYM theory on spatial $S^5 \times$ time \mathbf{R} .

Since all of the A baryons preserve the supersymmetries, there is no force between the baryon vertices. This means that we do not have any bound state forming the nucleus, due to the supersymmetries. Rather to say, we may consider an arbitrary distribution of baryon vertices. So, as an example, let us distribute the baryon vertices uniformly on an S^3 in the AdS_5 for simplicity. Since each bulk location of the component baryon vertex corresponds to the size and the location of the baryon

in the boundary SYM, this spherical distribution amounts to a ball-like distribution of baryons in the SYM (see Fig. 3). We chose this distribution by two reasons: First, it mimics real nuclei, the simplest among which have the shape of a ball. Second, with this spherical distribution of the baryon vertices, it is easy to compute a fluctuation spectrum in the geometry created by the baryon vertices.

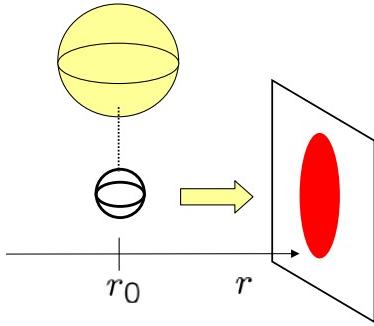


Figure 3: The baryon vertices are distributed on a sphere. The arrow denotes a projection onto the boundary space on the left.

As we described, collective excitations on a nucleus can be obtained by looking at a near horizon geometry of the D5 black-brane solution, in the large A limit. In general, computation of a fully back-reacted solution in the type IIB supergravity is quite complicated, hence we consider the following two simplifications. First, we need to see only the near-horizon geometry of the D5-branes for our purpose, thus we do not need the full geometry which asymptotes to the $AdS_5 \times S^5$ geometry. Second, we may ignore the presence of the spikes (= fundamental strings attached to the D5-branes in Fig. 2) and the electric charges on the D5-branes, by the following two reasons. The electric charge

density induced on the D5-brane worldvolume is proportional to $N/\lambda^{5/4}$, where N comes from the 5-form flux penetrating the S^5 as explained around (2.1) and $\lambda^{5/4}$ is the volume of the S^5 . So, in an appropriate limit $N, \lambda \rightarrow \infty$, this charge density can be arbitrarily small. Furthermore, in Fig. 2, the left hand side of the S^5 of the D5-brane seems not really affected by the fundamental strings which are attached to the right hand side of the S^5 of the D5-brane. Hence we may just consider only the D5-branes while ignoring the fundamental strings (and also the background flux which generates the strings). This second simplification would be an assumption, as we will not show in this paper that dynamics of the fundamental strings is irrelevant.

The first simplification can be realized by looking at the original $AdS_5 \times S^5$ metric around the position of the baryon vertex $r = r_0$,

$$ds^2 = \frac{r_0^2}{\alpha' \sqrt{2\lambda}} dx_{||}^2 + \frac{\alpha' \sqrt{\lambda}}{r_0} dr^2 + dy^2, \quad (2.3)$$

where $x_{||}$ means x^0, \dots, x^3 which are the spacetime coordinates for the $\mathcal{N} = 4$ SYM, and dy^2 is the metric on the S^5 . This large S^5 can be approximated by \mathbf{R}^5 for large N . Resultantly, if we make the following rescaling of the coordinates,

$$\tilde{x}_{||} \equiv \frac{r_0}{\sqrt{\alpha' \sqrt{2\lambda}}} x_{||}, \quad \tilde{r} \equiv \frac{\sqrt{\alpha' \sqrt{2\lambda}}}{r_0} r, \quad (2.4)$$

the metric is nothing but just the flat spacetime metric in 10 dimensions. In these rescaled coordinates, we can easily obtain a supergravity solution with a full back-

reaction of the flat A D5-branes (if we ignore the effect of the fundamental strings as explained before as the second simplification),

$$ds^2 = f^{-1/2}(-d\tilde{t}^2 + dy^2) + f^{1/2}(d\tilde{x}_i^2 + d\tilde{r}^2), \quad (2.5)$$

$$e^{-2\phi} = f. \quad (2.6)$$

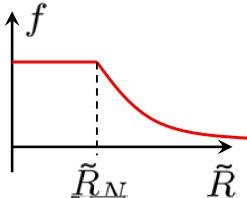


Figure 4: The geometry of back-reacted D5-branes distributed on the S^3 .

For the case of A D5-branes on top of each other, the harmonic function f is given by

$$f = 1 + \frac{g_s \alpha' A}{\tilde{R}^2} \quad (2.7)$$

where $\tilde{R} \equiv \sqrt{\tilde{x}_1^2 + \tilde{x}_2^2 + \tilde{x}_3^2 + \tilde{r}^2}$ is the radial distance from the coincident D5-branes in the rescaled coordinates.

Our interest is the D5-branes distributed (smeared) uniformly on the S^3 . Let us suppose that the radius of the shell S^3 is \tilde{R}_N in the rescaled coordinates. Then the function f is given by (2.7) for $\tilde{R} > \tilde{R}_N$, while f inside the shell is a constant, see Fig. 4. The continuity condition at the shell $\tilde{R} = \tilde{R}_N$ gives

$$f = 1 + \frac{g_s \alpha' A}{\tilde{R}_N^2} \quad (\tilde{R} < \tilde{R}_N). \quad (2.8)$$

The near horizon geometry with large A is simply given by ignoring “1+” in (2.7) and (2.8). This is the back-reacted near-horizon geometry which we need.

3. Giant resonance of heavy nuclei

According to the dictionary of the gauge/string duality, the supergravity fluctuation spectrum in this geometry corresponds to the spectrum of the theory on the D5-branes, that is, the spectrum of the heavy nucleus. The simplest fluctuation among supergravity fields is the dilaton fluctuation $\delta\phi$ whose equation of motion is*

$$\partial_M \left[\sqrt{-\det g^{(E)}} g^{(E)MN} \partial_N \delta\phi \right] = 0. \quad (3.1)$$

Here note that the metric is in the Einstein frame, $g_{MN}^{(E)} = e^{\phi/2} g_{MN}$. The standard dictionary tells us that $\delta\phi$ corresponds partly to an operator $\text{tr}[\Phi, \Phi]^2$ in the $U(A)$ SYM theory on the A D5-branes. This operator describes a certain collective motion of the constituent baryons, as the eigenvalues of the scalar field Φ on the D5-branes denote the location of the D5-branes.

*We assume that the fluctuation of the dilaton in the Einstein frame and a fluctuation of the Ramond-Ramond tensor fields are decoupled from each other.

To solve this equation (3.1) in our background, we consider the form

$$\delta\phi(\tilde{x}, \tilde{r}, y) = e^{i\tilde{E}\tilde{t}}g(\tilde{R}), \quad (3.2)$$

then (3.1) reduces to an ordinary differential equation

$$\left[\tilde{R}^3 f(\tilde{R}) \tilde{E}^2 + \partial_{\tilde{R}} \tilde{R}^3 \partial_{\tilde{R}} \right] g(\tilde{R}) = 0. \quad (3.3)$$

With a new variable $s \equiv 1/\tilde{R}^2$, this can be cast into a form of a Schrödinger equation, which can be solved. A simple dimensional analysis gives the energy

$$\tilde{E}^2 = \frac{c}{g_s \alpha' A} \quad (3.4)$$

where c is a dimensionless numerical constant.

Interestingly, the fluctuation spectrum in this background was already computed in [8]. It was shown that c in (3.4) is given by $c = 1$, and it is merely a gap, above which the spectrum is continuous. The motivation of [8] was quite different in origin: the S^3 spherical distribution was introduced there as a consistent UV cut-off for the gravity dual of the D5/NS5 theory to probe little string theory.

The mass gap (3.4) with $c = 1$ is written in the rescaled coordinates, thus bringing it to the original coordinates, we find

$$E = \tilde{E} \frac{r_0}{\sqrt{\alpha' \sqrt{2\lambda}}} = \frac{(2\lambda)^{1/4}}{z_0} A^{-1/2}. \quad (3.5)$$

Here in the last equality we have used (2.2). This expression (3.5) is the mass gap in the spectrum of the collective motion of the scalar excitation on the supersymmetric heavy nucleus. It should be called a monopole (spin zero) giant resonance of the supersymmetric nucleus.

In the following, we list key points of our result (3.5).

- We find an interesting A dependence in (3.5). The observed monopole giant resonances of heavy nuclei has $A^{-1/3}$ scaling, and our A -dependence is different. However, we will see in the next section that a similar analysis in type IIA string theory gives $A^{-1/3}$. We emphasize that our framework is capable of computing the A dependence — it is quite important that we can compare it with the A dependence observed in nature. This is contrasted with the standard N dependence of large N QCD theories where real QCD is fixed to be at $N = 3$.
- The gap (3.5) is independent of R_N but depends on z_0 . R_N is roughly the size of the nucleus, while z_0 is the size of each baryon. In nature, both of these should be determined by self-interactions of/among baryons, hence the dependence found in (3.5) would be due to the supersymmetries.
- The gap (3.5) does not depend on α' , due to (2.2) and the rescaling (2.4). All the physical quantities computed by the gauge/string duality should be independent of α' , and our interpretation is consistent.

4. Analysis in type IIA string theory

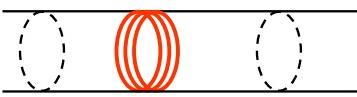


Figure 5: N D4-branes wrapping a circle.

We can perform a similar analysis for type IIA brane configurations. To form a $SU(N)$ QCD-like gauge theory, let us consider N D4-branes wrapping a circle, as in Fig. 5. The radius of the circle is chosen to be $1/M_{\text{KK}}$ so that the Kaluza-Klein (KK) modes on the circle has the mass scale M_{KK} .

At low energy on the D4-branes, all the KK modes are massive and decoupled, then the theory there becomes the 3+1-dimensional $\mathcal{N} = 4$ SYM theory. The gauge coupling is given by the circle compactification as

$$\frac{1}{g_{\text{YM}}^2} = \frac{2\pi}{M_{\text{KK}}} \frac{1}{(\pi)^2 g_s l_s}. \quad (4.1)$$

The gravity dual of this geometry is a near horizon geometry of a BPS black 4-brane solution of the type IIA supergravity.

Let us put baryon vertices which are D4-branes wrapping the S^4 of the geometry. As before, we specify the radial location of the baryon vertices as $r_0 = \alpha' U_0$ where U_0 is some physical scale, mimicking the relation (2.2). This U_0 is a free parameter.

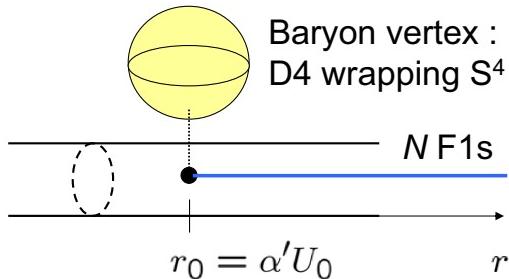


Figure 6: The brane configuration in the type IIA string theory.

Then, we distribute A baryon vertices on a certain S^4 around $r = r_0$ in the space transverse to the S^4 of the background geometry. This amounts to a procedure of having the ball-like distribution of baryons in 3-dimensional space of the SYM theory, as one can see by projecting the S^4 distribution of the D4-branes onto the boundary space.

We consider a near horizon geometry of the A D4-branes (baryon vertices) and take a large A limit. A similar analysis gives, this time, a discrete spectrum of the dilaton fluctuation,

$$\tilde{E} = \frac{c}{3} \sqrt{\frac{\tilde{R}_N}{\pi l_s^3 \tilde{g}_s A}}, \quad (4.2)$$

with discrete values $c = 2.23, 3.68, 4.75, \dots$. Here, again, \tilde{R}_N is the radius of the S^4 on which the D4-branes are distributed, in the rescaled coordinates. A similar rescaling back, $\tilde{E} \rightarrow E$, $\tilde{R}_N \rightarrow R_N$, $\tilde{g}_s \rightarrow g_s$, results in

$$E = \left[\frac{8M_{\text{KK}}^3 U_0^3}{N} \right]^{1/4} \frac{1}{3} R_N^{1/2} c A^{-1/2}. \quad (4.3)$$

With the discrete values of c , we find a spectrum of a giant resonance. Again, (4.3) does not depend on the string length $l_s^2 = \alpha'$, so it is a consistent observable in the gauge/string duality.

Let us study the A dependence. If we substitute by hand $R_N \propto A^{1/3}$ which we adopt since it is expected from the nuclear density saturation (an observed fact for realistic nuclei), we obtain

$$E \propto A^{-1/3}. \quad (4.4)$$

Interestingly, this dependence on A is in accordance with the observed monopole giant resonances.

5. Summary and Discussions

We have provided supersymmetric examples of a gravity dual of a heavy nucleus. The dual is the near horizon limit of the back-reacted geometry of the baryon vertices. The computation of the fluctuation spectra in this geometry gives the holographic derivation of spectra of giant resonances of heavy nuclei.

Generically, computations of supergravity backgrounds back-reacted due to the baryon vertices are complicated (see [9] for the case of spatially smeared baryons), but we find a simplification which leads to a known geometry of distributed black branes. There are a lot of known geometries of this kind, and they would be of importance for the application to nuclear physics, through the idea presented in this article and our previous paper [4], the holographic nuclei. On the other hand, in the gauge/string duality, important back-reacted geometries have been constructed (for example see [10]), for example for large number of fundamental strings in the bulk [11]. It would be interesting to explore possible relations between those geometries and the holographic nuclei.

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